CHAPTER 10
Fluids

Units

- Fluids in Motion; Flow Rate and the Equation of Continuity
- Bernoulli’s Equation
- Applications of Bernoulli’s Principle
- Viscosity
- Flow in Tubes: Poiseuille’s Equation, Blood Flow
- Surface Tension and Capillarity

Phases of Matter

The three common phases of matter are solid, liquid, and gas.
A solid has a definite shape and size.
A liquid has a fixed volume but can be any shape.
A gas can be any shape and also can be easily compressed.
Liquids and gases both flow, and are called fluids.

States of Matter

- Solid
- Liquid
- Gas
- Plasma

Liquid

- Has a definite volume
- No definite shape
- Exists at a higher temperature than solids
- The molecules “wander” through the liquid in a random fashion
  - The intermolecular forces are not strong enough to keep the molecules in a fixed position

Plasma

- Matter heated to a very high temperature
- Many of the electrons are freed from the nucleus
- Result is a collection of free, electrically charged ions
- Plasmas exist inside stars
Density and Specific Gravity

The density \( \rho \) of an object is its mass per unit volume:

\[
\rho = \frac{m}{V}
\]

The SI unit for density is kg/m\(^3\). Density is also sometimes given in g/cm\(^3\); to convert g/cm\(^3\) to kg/m\(^3\), multiply by 1000.

Water at 4°C has a density of 1 g/cm\(^3\) = 1000 kg/m\(^3\).

The specific gravity of a substance is the ratio of its density to that of water.

**Figure 1:** A cubical box 20.0 cm on a side is completely immersed in a fluid. At the top of the box the pressure is 105.0 kPa; at the bottom the pressure is 106.8 kPa. What is the density of the fluid?

\[
\rho = \frac{P_2 - P_1}{gh} \]

\[
\rho = \frac{1.068 \times 10^5 \text{ Pa} - 1.050 \times 10^5 \text{ Pa}}{(9.80 \text{ m/s}^2)(0.200 \text{ m})}
\]

\[
= 920 \text{ kg/m}^3
\]

**Example 2:** A water bed is 2.00 m on a side and 30.0 cm deep.

(a) Find its weight.

\[
V = lwh = (2.00 \text{ m})(2.00 \text{ m})(0.300 \text{ m}) = 1.20 \text{ m}^3
\]

\[
\rho = \frac{M}{V} \rightarrow M = \rho V = (1.00 \times 10^3 \text{ kg/m}^3)(1.20 \text{ m}^3) = 1.20 \times 10^3 \text{ kg}
\]

\[
w = Mg = (1.20 \times 10^3 \text{ kg})(9.80 \text{ m/s}^2) = 1.18 \times 10^4 \text{ N}
\]

(b) Find the pressure that the water bed exerts on the floor.

\[
P = \frac{F}{A} = \frac{w}{A} = \frac{1.18 \times 10^4 \text{ N}}{4.00 \text{ m}^2} = 2.95 \times 10^3 \text{ Pa}
\]

**Pressure in Fluids**

Pressure is defined as the force per unit area. Pressure is a scalar; the units of pressure in the SI system are pascals: 1 Pa = 1 N/m\(^2\). Pressure is the same in every direction in a fluid at a given depth; if it were not, the fluid would flow.
Example 3: the two feet of a 60-kg person cover an area of 500 cm². Determine the pressure exerted by the two feet on the ground.

\[ P = \frac{F}{A} = \frac{mg}{A} = \frac{(60 \text{ kg})(9.80 \text{ m/s}^2)}{(0.050 \text{ m}^2)} = 12 \times 10^3 \text{ N/m}^2 \]

**Pressure in Fluids**

Also for a fluid at rest, there is no component of force parallel to any solid surface – once again, if there were the fluid would flow.

The pressure at a depth \( h \) below the surface of the liquid is due to the weight of the liquid above it. We can quickly calculate:

\[ P = \rho gh \]

This relation is valid for any liquid whose density does not change with depth.

Example 4: the surface of the water in a storage tank is 30 m above a water faucet in the kitchen of a house. Calculate the difference in water pressure between the faucet and the surface of the water in the tank.

\[ \Delta P = \rho g \Delta h = (1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(30 \text{ m}) \]

\[ = 2.9 \times 10^5 \text{ N/m}^2 \]

**Pressure and Depth**

- Examine the darker region, assumed to be a fluid
  - It has a cross-sectional area \( A \)
  - Extends to a depth \( h \) below the surface
- Three external forces act on the region

**Pressure and Depth equation**

\[ P = P_o + \rho gh \]

- \( P_o \) is normal atmospheric pressure
  - \( 1.013 \times 10^5 \text{ Pa} = 14.7 \text{ lb/in}^2 \)
- The pressure does not depend upon the shape of the container
Static Equilibrium in Fluids: Pressure and Depth

The increased pressure as an object descends through a fluid is due to the increasing mass of the fluid above it.

\[ F_{\text{top}} = P_{\text{at}} A \]

\[ P_{\text{bottom}} = P_{\text{at}} + \rho gh \]

**Example 5:** The Titanic was found in 1985 lying on the bottom of the North Atlantic at a depth of 4022 m (2.5 mi). What is the pressure at this depth?

\( \rho = 1025 \text{ kg/m}^3 \)

\[ P = P_{\text{at}} + \rho gh \]

\[ = 1.01 \times 10^5 \text{ Pa} + (1025 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1609 \text{ m/1mi}) \]

\[ = 4.1 \times 10^7 \text{ Pa} \]

This is about 400 atmospheres

**Pascal's Principle**

- A change in pressure applied to an enclosed fluid is transmitted undiminished to every point of the fluid and to the walls of the container.
  - First recognized by Blaise Pascal, a French scientist (1623 – 1662)

**Atmospheric Pressure and Gauge Pressure**

At sea level the atmospheric pressure is about 1.013 \( \times 10^5 \text{ N/m}^2 \); this is called one atmosphere (atm).

Another unit of pressure is the bar:

\[ 1 \text{ bar} = 1.00 \times 10^5 \text{ N/m}^2 \]

Standard atmospheric pressure is just over 1 bar.

This pressure does not crush us, as our cells maintain an internal pressure that balances it.

Most pressure gauges measure the pressure above the atmospheric pressure – this is called the gauge pressure.

The absolute pressure is the sum of the atmospheric pressure and the gauge pressure.

\[ P = P_A + P_G \]
Pascal’s Principle

If an external pressure is applied to a confined fluid, the pressure at every point within the fluid increases by that amount. This principle is used, for example, in hydraulic lifts and hydraulic brakes.

Absolute vs. Gauge Pressure

- The pressure $P$ is called the **absolute** pressure
  - Remember, $P = P_0 + \rho gh$
- $P - P_0 = \rho gh$ is the **gauge** pressure

Measurement of Pressure; Gauges and the Barometer

There are a number of different types of pressure gauges. This one is an open-tube manometer. The pressure in the open end is atmospheric pressure; the pressure being measured will cause the fluid to rise until the pressures on both sides at the same height are equal.

Here are two more devices for measuring pressure: the aneroid gauge and the tire pressure gauge.
This is a mercury barometer, developed by Torricelli to measure atmospheric pressure. The height of the column of mercury is such that the pressure in the tube at the surface level is 1 atm. Therefore, pressure is often quoted in millimeters (or inches) of mercury.

**Pressure Values in Various Units**

- One atmosphere of pressure is defined as the pressure equivalent to a column of mercury exactly 0.76 m tall at 0°C where $g = 9.80665 \text{ m/s}^2$
- One atmosphere (1 atm) =
  - 76.0 cm of mercury
  - $1.013 \times 10^5 \text{ Pa}$
  - $14.7 \text{ lb/in}^2$

Any liquid can serve in a Torricelli-style barometer, but the most dense ones are the most convenient. This barometer uses water.

**Archimedes**

- 287 – 212 BC
- Greek mathematician, physicist, and engineer
- Buoyant force
- Inventor
Archimedes' Principle

- Any object completely or partially submerged in a fluid is buoyed up by a force whose magnitude is equal to the weight of the fluid displaced by the object.

Buoyant Force

- The magnitude of the buoyant force always equals the weight of the displaced fluid
  \[ B = \rho_{\text{fluid}} V_{\text{fluid}} g = w_{\text{fluid}} \]
- The buoyant force is the same for a totally submerged object of any size, shape, or density
- The buoyant force is exerted by the fluid
- Whether an object sinks or floats depends on the relationship between the buoyant force and the weight

This is an object submerged in a fluid. There is a net force on the object because the pressures at the top and bottom of it are different. The buoyant force is found to be the upward force on the same volume of water:

\[
F_B = F_2 - F_1 = \rho_F g A (h_2 - h_1) = \rho_F g A \Delta h = \rho_F V g = m_F g,
\]

Example 6: A 70-kg ancient statue lies at the bottom of the sea. Its volume is $3.0 \times 10^4 \text{cm}^3$. How much force is needed to lift it?

\[
F_B = m_{H_2O} g = \rho_{H_2O} V g = (1025 \text{kg/m}^3)(3.0 \times 10^{-2} \text{m}^3)(9.80 \text{m/s}^2) = 300 \text{N}
\]
Buoyancy and Archimedes’ Principle

The net force on the object is then the difference between the buoyant force and the gravitational force.

**Example 7:** When a crown of mass 14.7 kg is submerged in water, an accurate scale reads 13.4 kg. Is the crown made of gold?

\[
\rho_{\text{obj}} = \frac{w}{w' - \rho_{\text{H}_2\text{O}}} = \frac{14.7\text{kg} \cdot g}{(14.7\text{kg} - 13.4\text{kg}) \cdot g} = \frac{14.7\text{kg}}{1.3\text{kg}} = 11.3
\]

This corresponds to a density of 11,300 kg/m³. The crown seems to be made of lead.

If the object’s density is less than that of water, there will be an upward net force on it, and it will rise until it is partially out of the water.

For a floating object, the fraction that is submerged is given by the ratio of the object's density to that of the fluid.
Example 8: A raft is constructed of wood having a density of \(6.00 \times 10^2 \text{ kg/m}^3\). Its surface area is \(5.70 \text{ m}^2\) and its volume is \(0.60 \text{ m}^3\). When the raft is placed in fresh water to what depth \(h\) is the bottom of the raft submerged?

\[
B - m_{\text{raft}} g = 0 \rightarrow B = m_{\text{raft}} g \\
B = m_{\text{water}} g = (\rho_{\text{water}} V_{\text{water}}) g = (\rho_{\text{water}} Ah) g \\
m_{\text{raft}} g = (\rho_{\text{raft}} V_{\text{raft}}) g \\
(\rho_{\text{water}} Ah) g = (\rho_{\text{raft}} V_{\text{raft}}) g \\

h = \frac{\rho_{\text{raft}} V_{\text{raft}}}{\rho_{\text{water}} A} = \frac{(6.00 \times 10^2 \text{ kg/m}^3)(0.600 \text{ m}^3)}{(1.00 \times 10^3 \text{ kg/m}^3)(5.70 \text{ m}^2)} = 0.0632 \text{ m}
\]

Fluids in Motion; Flow Rate and the Equation of Continuity

If the flow of a fluid is smooth, it is called streamline or laminar flow (a).

Above a certain speed, the flow becomes turbulent (b). Turbulent flow has eddies; the viscosity of the fluid is much greater when eddies are present.

We will deal with laminar flow.

The mass flow rate is the mass that passes a given point per unit time. The flow rates at any two points must be equal, as long as no fluid is being added or taken away.

This gives us the equation of continuity:

\[
\rho_1 A_1 v_1 = \rho_2 A_2 v_2
\]

If the density doesn’t change – typical for liquids – this simplifies to \(A_1 v_1 = A_2 v_2\). Where the pipe is wider, the flow is slower.

Characteristics of an Ideal Fluid

- The fluid is nonviscous
  - There is no internal friction between adjacent layers
- The fluid is incompressible
  - Its density is constant
- The fluid motion is steady
  - Its velocity, density, and pressure do not change in time
- The fluid moves without turbulence
  - No eddy currents are present
Example 9: A water hose 2.5 cm in diameter is used by a gardener to fill a 30.0-liter bucket. (One liter = 1000 cm$^3$.) The hose takes 1.00 min to fill the bucket. A nozzle with an opening of cross-sectional area 0.500 cm$^2$ is then attached to the hose. The nozzle is held so that water is projected horizontally from a point 1.00 m above the ground. Over what horizontal distance can the water be projected?

Calculate the volume flow rate into the bucket, and convert to $m^3 / s$:

\[
\text{volume flow rate} = \frac{30.0\text{L}}{1.00\text{min}} \left( \frac{1.00 \times 10^3 \text{cm}^3}{1.00 \text{L}} \right) \left( \frac{1.00\text{m}}{100.0\text{cm}} \right)^3 \left( \frac{1.00\text{min}}{60.0\text{s}} \right) = 5.00 \times 10^{-4} \text{m}^3 / \text{s}
\]

\[A_1v_1 = A_2v_2 = A_2v_{ox} \quad v_{ox} = \frac{A_1v_1}{A_2} = \frac{5.00 \times 10^{-4} \text{m}^3 / \text{s}}{0.500 \times 10^{-4} \text{m}^2} = 10.0 \text{m/s}
\]

\[\Delta y = v_{ox} t - \frac{1}{2} gt^2 \quad t = \sqrt{\frac{-2\Delta y}{g}} = \sqrt{\frac{-2(-1.00\text{m})}{9.80 / \text{m/s}^2}} = 0.452\text{s}
\]

\[x = v_{ox} t = (10.0 \text{m/s})(0.452\text{s}) = 4.52\text{m}
\]

Bernoulli’s Equation

A fluid can also change its height. By looking at the work done as it moves, we find:

\[P + \frac{1}{2} \rho v^2 + \rho gy = \text{constant}
\]

This is Bernoulli’s equation. One thing it tells us is that as the speed goes up, the pressure goes down.

Equation of Continuity

- $A_1v_1 = A_2v_2$
- The product of the cross-sectional area of a pipe and the fluid speed is a constant
  - Speed is high where the pipe is narrow and speed is low where the pipe has a large diameter
- $Av$ is called the flow rate
• The equation is a consequence of conservation of mass and a steady flow
• \( A_1 v = \text{constant} \)
  – This is equivalent to the fact that the volume of fluid that enters one end of the tube in a given time interval equals the volume of fluid leaving the tube in the same interval
  • Assumes the fluid is incompressible and there are no leaks

**Example 10:** A large pipe with a cross-sectional area of \(1.00 \text{m}^2\) descends 5.00 m and narrows to \(0.500 \text{m}^2\) where it terminates in a valve. If the pressure at point (2) is atmospheric pressure, and the valve is opened wide and water allowed to flow freely, find the speed of the water leaving the pipe.

\[
P_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \quad A_2 v_2 = A_1 v_1 \quad v_2 = \frac{A_1}{A_2} v_1
\]

\[
P_0 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_0 + \frac{1}{2} \rho \left( \frac{A_1}{A_2} v_1 \right)^2 + \rho p g y_2 \quad v_1^2 \left( 1 - \left( \frac{A_1}{A_2} \right)^2 \right) = 2 g (y_2 - y_1) = 2 g h
\]

\[
v_1 = \frac{\sqrt{2 g h}}{\sqrt{1 - (A_1 / A_2)^2}} = 11.4 \text{m/s}
\]

When a fluid moves from a wider area of a pipe to a narrower one, its speed increases; therefore, work has been done on it.

\[
\Delta W_{\text{total}} = (P_1 - P_2) \Delta V
\]

The kinetic energy of a fluid element is:

\[
K = \frac{1}{2} (\Delta m) v^2 = \frac{1}{2} (\rho \Delta V) v^2
\]

Equating the work done to the increase in kinetic energy gives:

\[
P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2
\]
If a fluid flows in a pipe of constant diameter, but changes its height, there is also work done on it against the force of gravity. Equating the work done with the change in potential energy gives:

\[ P_1 + \rho gy_1 = P_2 + \rho gy_2 \]

The general case, where both height and speed may change, is described by Bernoulli’s equation:

**Bernoulli’s Equation**

\[ P_1 + \frac{1}{2} \rho v_1^2 + \rho gy_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho gy_2 \]

This equation is essentially a statement of conservation of energy in a fluid.

**Daniel Bernoulli**
- 1700 – 1782
- Swiss physicist and mathematician
- Wrote *Hydrodynamica*
- Also did work that was the beginning of the kinetic theory of gases

**Applications of Bernoulli’s Principle:**

Using Bernoulli’s principle, we find that the speed of fluid coming from a spigot on an open tank is:

\[ v_1 = \sqrt{2g(y_2 - y_1)} \]

This is called Torricelli’s theorem.

If a hole is punched in the side of an open container, the outside of the hole and the top of the fluid are both at atmospheric pressure. Since the fluid inside the container at the level of the hole is at higher pressure, the fluid has a horizontal velocity as it exits.
If the fluid is directed upwards instead, it will reach the height of the surface level of the fluid in the container.

**Example 11**: A water tank has a hole in it causing it to leak.
(a) If the top of the tank is open to the atmosphere, determine the speed at which the water leaves the hole when the water level is 0.500 m above the hole.

\[
P_o + \frac{1}{2} \rho v_1^2 + \rho g y_1 = P_o + \rho g y_2
\]

\[
v_1 = \sqrt{2g(y_2 - y_1)} = \sqrt{2gh}
\]

\[
v_1 = \sqrt{2(9.80 \text{m/s}^2)(0.500 \text{m})} = 3.13 \text{m/s}
\]

(b) Where does the stream hit the ground if the hole is 3.00 m above the ground?

\[
\Delta y = -\frac{1}{2} gt^2 + v_{oy} t
\]

\[
-3.00 \text{m} = -(4.90 \text{m/s}^2)t^2 \quad t = 0.782 \text{s} \quad x = v_{oy} t = (3.13 \text{m/s})(0.782 \text{s}) = 2.45 \text{m}
\]

A venturi meter can be used to measure fluid flow by measuring pressure differences.

**Viscosity**

Real fluids have some internal friction, called viscosity. The viscosity can be measured; it is found from the relation

\[
F = \eta A \frac{v}{l}
\]

where \( \eta \) is the coefficient of viscosity.
Flow in Tubes; Poiseuille’s Equation

The rate of flow in a fluid in a round tube depends on the viscosity of the fluid, the pressure difference, and the dimensions of the tube. The volume flow rate is proportional to the pressure difference, inversely proportional to the length of the tube and to the pressure difference, and proportional to the fourth power of the radius of the tube.

Surface Tension and Capillarity

The surface of a liquid at rest is not perfectly flat; it curves either up or down at the walls of the container. This is the result of surface tension, which makes the surface behave somewhat elastically.

Soap and detergents lower the surface tension of water. This allows the water to penetrate materials more easily.

Water molecules are more strongly attracted to glass than they are to each other; just the opposite is true for mercury.

If a narrow tube is placed in a fluid, the fluid will exhibit capillarity.
1. A piece of iron sinks to the bottom of a lake where the pressure is 21 atm. The volume of that piece of iron decreases slightly.

2. A piece of iron sinks to the bottom of a lake where the pressure is 21 atm. The density of that piece of iron increases slightly.

3. When soup gets cold, it often tastes greasy. This “greasy” taste seems to be associated with oil spreading out all over the surface of the soup, instead of staying in little globs. This is explained in terms of the increase in the surface tension of water with increasing temperature.

4. The greatest effect on the flow of fluid through a narrow pipe when a 10% change in the fluid viscosity, length of pipe, the pressure difference and radius of the pipe is the radius.

5. Three drinking glasses have the same area base, and all three are filled to the same depth with water. Glass A is cylindrical, Glass B is wider at the top than at the bottom, and so holds more than A. Glass C is narrower at the top than at the bottom, and so holds less water than A. All three glasses have the same pressure.

6. When atmospheric pressure changes, the absolute pressure at the bottom of a pool will increase by the same amount.

7. As a rock sinks deeper and deeper into water of constant density, the buoyant force on it remains constant.

8. 50 cm$^3$ of wood is floating on water, and 50 cm$^3$ of iron is totally submerged. The greater buoyant force is on the iron.

9. A piece of iron rests on top of a piece of wood floating in a bathtub. If the iron is removed from the wood, the water in the tub will go down.
10. A piece of wood is floating in a bathtub. A second piece of wood sits on top of the first piece, and does not touch the water. If the top piece is taken off and placed in the water, the water level in the tub does not change.

11. Salt water is more dense than fresh water. A ship floats in both fresh water and salt water. Compared to the fresh water, the amount of water displaced in the salt water is less.

12. A steel ball sinks in water but floats in a pool of mercury. The buoyant force on the ball is greater floating on the mercury.

13. As the speed of a moving fluid increases, the pressure in the fluid decreases.

14. The difference between the pressures inside and outside a tire is called gauge pressure.

15. Water flows through a pipe. The diameter of the pipe at point 1 is larger than at point 2. The speed of the water is greater at point 2.

16. Water flows through a pipe. The diameter of the pipe at point 1 is larger than at point 2. The water pressure is the greatest at point 1.