CHAPTER 8
Rotational Motion

Units

- Angular Quantities
- Constant Angular Acceleration
- Rolling Motion (Without Slipping)
- Torque
- Rotational Dynamics; Torque and Rotational Inertia
- Solving Problems in Rotational Dynamics

This topic will deal with rotational motion. This will involve the kinematics of rotational motion and then its dynamics (involving torque), as well as rotational kinetic energy and angular momentum. Only the rotation of rigid objects will be discussed. A **rigid object** is an object with a definite shape that doesn’t change, so that the particles composing it stay in fixed positions relative to another.

**Angular Quantities**

In purely rotational motion, all points on the object move in circles around the axis of rotation (“O”). The radius of the circle is \( r \). All points on a straight line drawn through the axis move through the same angle in the same time. The angle \( \theta \) in radians is defined:

\[
\theta = \frac{l}{r}
\]

where \( l \) is the arc length.

**The Radian**

- The radian is a unit of angular measure
- The radian can be defined as the arc length \( s \) along a circle divided by the radius \( r \)

\[
\theta = \frac{s}{r}
\]
Radians

- Comparing degrees and radians
  \[ 1 \text{ rad} = \frac{360^\circ}{2\pi} = 57.3^\circ \]
- Converting from degrees to radians
  \[ \theta [\text{rad}] = \frac{\pi}{180^\circ} \theta [\text{degrees}] \]

Example 1: A bike wheel rotates 4.50 revolutions. How many radians has it rotated?

1 revolution = 360° = 2\pi rad = 6.28 rad

4.50 revolutions = (4.50 rev) \left(\frac{2\pi \text{ rad}}{\text{rev}}\right) = 9.00\pi \text{ rad} = 28.3 \text{ rad}

Example 2: A particular bird’s eye can just distinguish objects that subtend an angle no smaller than about 3 \times 10^{-4} \text{ rad}.

(a) How many degrees is this?

\[(3 \times 10^{-4} \text{ rad}) \left(\frac{360^\circ}{2\pi \text{ rad}}\right) = 0.017^\circ\]

(b) How small an object can the bird just distinguish when flying at a height of 100 m?

\[l = (100 \text{ m})(3 \times 10^{-4} \text{ rad}) = 3 \times 10^{-2} \text{ m} = 3 \text{ cm}\]

Angular Quantities

Angular displacement: \[ \Delta \theta = \theta_2 - \theta_1 \]

The average angular velocity is defined as the total angular displacement divided by time:

\[ \bar{\omega} = \frac{\Delta \theta}{\Delta t} \]

The instantaneous angular velocity:

\[ \omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} \]
Average Angular Speed

- The average angular speed, \( \omega \), of a rotating rigid object is the ratio of the angular displacement to the time interval.

\[
\omega_{av} = \frac{\theta_f - \theta_i}{t_f - t_i} = \frac{\Delta \theta}{\Delta t}
\]

Angular Speed

- The instantaneous angular speed is defined as the limit of the average speed as the time interval approaches zero.
- Units of angular speed are radians/sec – rad/s.
- Speed will be positive if \( \theta \) is increasing (counterclockwise).
- Speed will be negative if \( \theta \) is decreasing (clockwise).

Every point on a rotating body has an angular velocity \( \omega \) and a linear velocity \( v \). They are related:

\[
v = r\omega
\]

Angular Quantities

Therefore, objects farther from the axis of rotation will move faster.
The angular acceleration is the rate at which the angular velocity changes with time:

$$\alpha = \frac{\omega_2 - \omega_1}{\Delta t} = \frac{\Delta \omega}{\Delta t}$$

The instantaneous acceleration:

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t}$$

**Angular Quantities**

If the angular velocity of a rotating object changes, it has a tangential acceleration:

$$a_{\tan} = r\alpha$$

Even if the angular velocity is constant, each point on the object has a centripetal acceleration:

$$a_R = \frac{v^2}{r} = \frac{(r\omega)^2}{r} = \omega^2 r$$

Here is the correspondence between linear and rotational quantities:

<table>
<thead>
<tr>
<th>Linear</th>
<th>Type</th>
<th>Rotational</th>
<th>Relation</th>
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</thead>
<tbody>
<tr>
<td>$x$</td>
<td>displacement</td>
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</tr>
<tr>
<td>$v$</td>
<td>velocity</td>
<td>$\omega$</td>
<td>$v = r\omega$</td>
</tr>
<tr>
<td>$a_{\tan}$</td>
<td>acceleration</td>
<td>$\alpha$</td>
<td>$a_{\tan} = r\alpha$</td>
</tr>
</tbody>
</table>

**Relationship between Angular and Linear Quantities**

- Displacements  $s = \theta r$
- Speeds  $v_{\ell} = \omega r$
- Accelerations  $a_{\ell} = \alpha r$

- Every point on the rotating object has the same angular motion
- Every point on the rotating object does **not** have the same linear motion
Angular Acceleration

- Units of angular acceleration are rad/s²
- Positive angular accelerations are in the counterclockwise direction and negative accelerations are in the clockwise direction
- When a rigid object rotates about a fixed axis, every portion of the object has the same angular speed and the same angular acceleration
- The sign of the acceleration does not have to be the same as the sign of the angular speed
- The instantaneous angular acceleration is defined as the limit of the average acceleration as the time interval approaches zero

Example 3: A wheel rotates with a constant angular acceleration of 3.5 rad/s². If the angular speed of the wheel is 2.00 rad/s at t = 0,
(a) through what angle does the wheel rotate between t = 0 and t = 2.00s?

\[
\Delta \theta = \omega t + \frac{1}{2} \alpha t^2
\]

\[
= (2.00 \text{rad/s})(2.00s) + \frac{1}{2} (3.50 \text{rad/s}^2)(2.00s)^2 = 11.0 \text{rad}
\]

\[
\Delta \theta = (11.0 \text{rad})(1.00 \text{rev/2\pi rad}) = 1.75 \text{rev}
\]

(b) What is the angular speed of the wheel at t = 2.00s?
\[
\omega = \omega_0 + \alpha t
\]

\[
= 2.00 \text{rad/s} + (3.50 \text{rad/s}^2)(2.00s) = 9.00 \text{rad/s}
\]

Example 4: An airplane propeller slows from an initial angular speed of 12.5 rev/s to a final angular speed of 5.00 rev/s. During this process, the propeller rotates through 21.0 revolutions. Find the angular acceleration of the propeller in radians per second squared.

\[
\Delta \theta = (21.0 \text{rev})(2\pi \text{rad/rev}) = 42.0\pi \text{rad}
\]

\[
\omega_i = (12.5 \text{rev/s})(2\pi \text{rad/rev}) = 25.0\pi \text{rad/s}
\]

\[
\omega = (5.00 \text{rev/s})(2\pi \text{rad/rev}) = 10.0\pi \text{rad/s}
\]

\[
\omega^2 = \omega_i^2 + 2\alpha \Delta \theta
\]

\[
(10.0\pi \text{rad/s})^2 = (25.0\pi \text{rad/s})^2 + 2\alpha(42\pi \text{rad}) = -6.25\pi \text{rad/s}^2
\]
Example 5: A carousel is initially at rest. At $t = 0$ it is given a constant angular acceleration $\alpha = 0.06 \text{rad} / s^2$, which increases its angular velocity of 8.0s. At $t = 8.0s$ determine the following quantities:

(a) The angular velocity of the carousel;

$$\omega_2 = \omega_1 + \alpha \Delta t$$

$$= 0 + (0.06 \text{rad} / s^2)(8.0s) = 0.48 \text{rad} / s$$

(b) The linear velocity of a child located 2.5 m from the center;

$$v = r\omega = (2.5m)(0.48 \text{rad} / s) = 1.2m / s$$

(c) The tangential (linear) acceleration of that child;

$$a_{tan} = r\alpha = (2.5m)(0.06 \text{rad} / s^2) = 0.15m / s^2$$

(d) The centripetal acceleration of the child;

$$a_R = \frac{v^2}{r} = \frac{(1.2m / s)^2}{(2.5m)} = 0.58m / s^2$$

(e) The total linear acceleration of the child.

$$a = \sqrt{a_{tan}^2 + a_R^2} = \sqrt{(0.15m / s^2)^2 + (0.58m / s^2)^2} = 0.60m / s^2$$

$$\theta = \tan^{-1}\left(\frac{a_{tan}}{a_R}\right) = \tan^{-1}\left(\frac{0.15m / s^2}{0.58m / s^2}\right) = 0.25\text{rad}$$

Angular Quantities

The frequency is the number of complete revolutions per second:

$$f = \frac{\omega}{2\pi}$$

Frequencies are measured in hertz.

$$1 \text{ Hz} = 1 \text{ s}^{-1}$$

The period is the time one revolution takes:

$$T = \frac{1}{f}$$
Example 6: the platter of a hard drive of a computer rotates at 7200 rpm. (a) What is the angular velocity of the platter?

\[ f = \frac{(7200 \text{ rev/min})}{(60 \text{ s/min})} = 120 \text{ rev/s} = 120 \text{ Hz} \]

\[ \omega = 2\pi f = 754 \text{ rad/s} \]

(b) If the read head of the drive is located 3.00 cm from the rotation axis, what is the linear speed of the point on the platter just below it?

\[ v = r\omega = (3.00 \times 10^{-2} \text{ m})(754 \text{ rad/s}) = 22.6 \text{ m/s} \]

(c) If a single bit requires 0.50 \text{ \mu m} of length along the direction of motion, how many bits per second can the write-head write when it is 3.00 cm from the axis?

\[ \frac{22.6 \text{ m/s}}{0.50 \times 10^{-6} \text{ m/bit}} = 45 \times 10^6 \text{ bits per second} \]

Example 7: A CD player read-head moves out from the center of the disc, the angular speed of the disc changes so that the linear speed at the position head remains at a constant value of about 1.3 m/s. (a) Find the angular speed of the CD when the read-head is at \( r = 2.0 \text{ cm} \).

\[ \omega = \frac{v_i}{r} = \frac{1.3 \text{ m/s}}{2.0 \times 10^{-2} \text{ m}} = 65 \text{ rad/s} \]

(b) A record player rotates at a constant angular speed, so the linear speed of the record groove moving under the stylus changes. Find the linear speed of a 45.0-rpm record at 2.0 cm from the center.

\[ 45.0 \text{ rev/min} = 45.0 \text{ rev/min} \left( \frac{2\pi \text{ rad}}{\text{ rev}} \right) \left( \frac{1.00 \text{ min}}{60.0 \text{ s}} \right) = 4.71 \text{ rad/s} \]

\[ v_i = r\omega = (2.0 \times 10^{-2} \text{ m})(4.71 \text{ rad/s}) = 0.094 \text{ m/s} \]

(c) In both the CD and phonograph record, information is recorded in a continuous spiral track. Calculate the total length for a CD designed to play for 2 hours.

\[ d = v_i t = (1.3 \text{ m/s})(7200 \text{ s}) = 9360 \text{ m} \]

Example 8: A centrifuge rotor is accelerated from rest to 20,000 rpm in 30s. (a) What is its average angular acceleration?

\[ \omega = 2\pi f = 2\pi \text{ rad/rev} \left( \frac{20,000 \text{ rev/min}}{60 \text{ s/min}} \right) = 2100 \text{ rad/s} \]

\[ \alpha = \frac{\omega - \omega_0}{\Delta t} = \frac{2100 \text{ rad/s} - 0}{30 \text{ s}} = 70 \text{ rad/s}^2 \]

\[ \frac{70}{2\pi} = 11 \text{ rev/sec} \]
(b) Through how many revolutions has the centrifuge rotor turned during its acceleration period?

\[ \theta = \omega t + \frac{1}{2} \alpha t^2 = 0 + \frac{1}{2} (70 \text{ rad} / \text{s}^2)(30 \text{s})^2 = 3.15 \times 10^4 \text{ rad} \]

\[ \frac{3.15 \times 10^4 \text{ rad}}{2 \pi \text{ rad} / \text{ rev}} = 5.0 \times 10^3 \text{ rev} \]

**Vector Nature of Angular Quantities**

- Angular displacement, velocity and acceleration are all vector quantities.
- Direction can be more completely defined by using the right hand rule:
  - Grasp the axis of rotation with your right hand
  - Wrap your fingers in the direction of rotation
  - Your thumb points in the direction of \( \omega \)

**Velocity Directions**

- In (a), the disk rotates clockwise, the velocity is into the page.
- In (b), the disk rotates counterclockwise, the velocity is out of the page.

**Acceleration Directions**

- If the angular acceleration and the angular velocity are in the same direction, the angular speed will increase with time.
- If the angular acceleration and the angular velocity are in opposite directions, the angular speed will decrease with time.
Forces Causing Centripetal Acceleration

- Newton’s Second Law says that the centripetal acceleration is accompanied by a force
  \[ F_C = ma_C \]
  - \( F_C \) stands for any force that keeps an object following a circular path
    - Tension in a string
    - Gravity
    - Force of friction

Constant Angular Acceleration

The equations of motion for constant angular acceleration are the same as those for linear motion, with the substitution of the angular quantities for the linear ones

<table>
<thead>
<tr>
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<th>Linear</th>
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<tbody>
<tr>
<td>( \omega = \omega_0 + \alpha t )</td>
<td>( v = v_0 + at )</td>
</tr>
<tr>
<td>( \theta = \omega_0 t + \frac{1}{2} \alpha t^2 )</td>
<td>( x = v_0 t + \frac{1}{2} at^2 )</td>
</tr>
<tr>
<td>( \omega^2 = \omega_0^2 + 2\alpha \theta )</td>
<td>( v^2 = v_0^2 + 2\alpha x )</td>
</tr>
<tr>
<td>( \bar{\omega} = \frac{\omega + \omega_0}{2} )</td>
<td>( \bar{v} = \frac{v + v_0}{2} )</td>
</tr>
</tbody>
</table>

Centripetal Acceleration

- An object traveling in a circle, even though it moves with a constant speed, will have an acceleration
- The centripetal acceleration is due to the change in the direction of the velocity
- Centripetal refers to “center-seeking”
- The direction of the velocity changes
- The acceleration is directed toward the center of the circle of motion
- The magnitude of the centripetal acceleration is given by
  \[ a_C = \frac{v^2}{r} \]
  - This direction is toward the center of the circle
Example 9: A race car accelerates uniformly from a speed of 40.0 m/s to a speed of 60.0 m/s in 5.00 s while traveling counterclockwise around a circular track of radius 400 m. When the car reaches a speed of 50.0 m/s, find:
(a) The magnitude of the car’s centripetal acceleration.

\[ a_c = \frac{v^2}{r} = \frac{(50.0 \text{ m/s})^2}{(400 \text{ m})} = 6.25 \text{ m/s}^2 \]

(b) The angular speed;

\[ \omega = \frac{v}{r} = \frac{50.0 \text{ m/s}}{400 \text{ m}} = 0.125 \text{ rad/s} \]

(c) The tangential acceleration;

\[ a_t = \frac{v_f - v_i}{\Delta t} = \frac{60.0 \text{ m/s} - 40.0 \text{ m/s}}{5.00 \text{ s}} = 4.00 \text{ m/s}^2 \]

(d) The magnitude of the total acceleration.

\[ a = \sqrt{a_t^2 + a_c^2} = \sqrt{(4.0 \text{ m/s}^2)^2 + (6.25 \text{ m/s}^2)^2} = 7.42 \text{ m/s}^2 \]

Rolling Motion (Without Slipping)

In (a), a wheel is rolling without slipping. The point P, touching the ground, is instantaneously at rest, and the center moves with velocity \( v \).

In (b) the same wheel is seen from a reference frame where C is at rest. Now point P is moving with velocity \( -v \).

The linear speed of the wheel is related to its angular speed:

\[ v = r \omega \]

Example 10: A bike slows down uniformly from \( v_o = 8.40 \text{ m/s} \) to rest over a distance of 11.5 m. Each wheel and tire has an overall diameter of 68.0 cm. Determine the following:
(a) the angular velocity of the wheels at the initial instant \( t = 0 \);

\[
\omega_o = \frac{v}{r} = \frac{8.40 \text{ m/s}}{0.340 \text{ m}} = 24.7 \text{ rad/s}
\]

(b) the total number of revolutions each wheel rotates before coming to rest;

\[
\frac{115 \text{ m}}{2\pi r} = \frac{115}{(2\pi)(0.340 \text{ m})} = 53.8 \text{ rev}
\]

(c) the angular acceleration of the wheel;

\[
\alpha = \frac{\omega^2 - \omega_o^2}{2\theta} = \frac{0 - (24.7 \text{ rad/s})^2}{2(2\pi \text{ rad/rev})(53.8 \text{ rev})} = -0.902 \text{ rad/s}^2
\]

(d) the time it took to come to a stop.

\[
t = \frac{\omega - \omega_o}{\alpha} = \frac{0 - 24.7 \text{ rad/s}}{-0.902 \text{ rad/s}^2} = 27.4 \text{ s}
\]

**Applications of Forces Causing Centripetal Acceleration**

- Many specific situations will use forces that cause centripetal acceleration
  - Level curves
  - Banked curves
  - Horizontal circles
  - Vertical circles

**Level Curves**

- Friction is the force that produces the centripetal acceleration
- Can find the frictional force, \( \mu \), or \( v \)

\[
v = \sqrt{\mu rg}
\]

**Banked Curves**

A component of the normal force adds to the frictional force to allow higher speeds

\[
\tan \theta = \frac{v^2}{rg}
\]

or \( a_c = g \tan \theta \)
Example 11: A race track has 31.0° banked curves, with maximum radius of 316 m. If a car negotiates the curve too slowly, it tends to slip down the incline of the turn, whereas if it's going too fast, it may begin to slide up the incline.

(a) Find the necessary centripetal acceleration on this banked curve so the car won’t slide up or down.

\[ a_c = g \tan \theta = (9.80 \text{ m/s}^2)(\tan 31.0°) = 5.89 \text{ m/s}^2 \]

(b) Calculate the speed of the race car.

\[ a_c = \frac{v^2}{r} \quad v = \sqrt{a_cr} \]

\[ v = \sqrt{(5.89 \text{ m/s}^2)(316 \text{ m})} = 43.1 \text{ m/s} \]

**Vertical Circle**

- Look at the forces at the top of the circle
- The minimum speed at the top of the circle can be found

\[ v_{top} = \sqrt{gR} \]

**Force vs. Torque**

- Forces cause accelerations
- Torques cause angular accelerations
- Force and torque are related

**Torque**

To make an object start rotating, a force is needed; the position and direction of the force matter as well. The perpendicular distance from the axis of rotation to the line along which the force acts is called the lever arm.
• The door is free to rotate about an axis through O
• There are three factors that determine the effectiveness of the force in opening the door:
  – The magnitude of the force
  – The position of the application of the force
  – The angle at which the force is applied

• Torque, \( \tau \), is the tendency of a force to rotate an object about some axis
  \[ \tau = r F \]
  • \( \tau \) is the torque
  • \( F \) is the force
  • symbol is the Greek tau
  • \( r \) is the length of the position vector

• SI unit is N\( \cdot \)m

**Direction of Torque**

• Torque is a vector quantity
  – The direction is perpendicular to the plane determined by the position vector and the force
  – If the turning tendency of the force is counterclockwise, the torque will be positive
  – If the turning tendency is clockwise, the torque will be negative

**Multiple Torques**

• When two or more torques are acting on an object, the torques are added
  – As vectors
• If the net torque is zero, the object’s rate of rotation doesn’t change

**General Definition of Torque**

• The applied force is not always perpendicular to the position vector
• The component of the force perpendicular to the object will cause it to rotate

• When the force is parallel to the position vector, no rotation occurs
• When the force is at some angle, the perpendicular component causes the rotation
• Taking the angle into account leads to a more general definition of torque:
  \[ \tau = r \cdot F \sin q \]
  • \( F \) is the force
  • \( r \) is the position vector
  • \( q \) is the angle between the force and the position vector

A longer lever arm is very helpful in rotating objects.

Here, the lever arm for \( F_A \) is the distance from the knob to the hinge; the lever arm for \( F_D \) is zero; and the lever arm for \( F_C \) is as shown.

The torque is defined as:

\[ \tau = r_\perp F \]

**Lever Arm**

• The lever arm, \( d \), is the perpendicular distance from the axis of rotation to a line drawn along the direction of the force
• \( d = r \sin q \)

**Torque and Equilibrium**

• First Condition of Equilibrium
  • The net external force must be zero

\[
\sum \vec{F} = 0 \quad \text{or} \\
\sum \vec{F}_x = 0 \quad \text{and} \quad \sum \vec{F}_y = 0
\]

  – This is a necessary, but not sufficient, condition to ensure that an object is in complete mechanical equilibrium
  – This is a statement of translational equilibrium

• To ensure mechanical equilibrium, you need to ensure rotational equilibrium as well as translational
• The Second Condition of Equilibrium states
  • The net external torque must be zero

\[
\sum \vec{r} = 0
\]

**Equilibrium Example**

• The woman, mass \( m \), sits on the left end of the seesaw
• The man, mass \( M \), sits where the seesaw will be balanced
• Apply the Second Condition of Equilibrium and solve for the unknown distance, \( x \)

**Example 12:** two people are pushing on a revolving door that has a diameter of 2.60 m. The person on the left exerts a force of 625 N perpendicular to the door and 1.2 m from the hub’s center, while the other one on the right exerts a force of 850 N perpendicular to the door and 0.800 m from the hub’s center. Find the net torque on the revolving door.

\[
\tau_1 = -r_1 F_1 = -(1.20m)(625N) = -750N \cdot m \\
\tau_2 = r_2 F_2 = (0.800m)(850N) = 680N \cdot m \\
\tau_{net} = \tau_1 + \tau_2 = -70N \cdot m
\]
Example 13: Two helmsmen, in disagreement about which way to turn a ship; exert the forces shown on a ship’s wheel. The wheel has a radius of 0.74m, and the two forces have magnitudes F1 = 72N and F2 = 58N.

(a) Find the torque caused by F1 and F2.

\[ \tau_1 = r F_1 \sin 50.0^\circ = (0.74m)(72N) \sin 50.0^\circ = 41N \cdot m \]

\[ \tau_2 = -r F_2 \sin 90.0^\circ = -(0.74m)(58N) = -43N \cdot m \]

(b) In which direction does the wheel turn as a result of these two forces?

\[ \tau_{net} = \tau_1 + \tau_2 = 41N \cdot m - 43N \cdot m = -2N \cdot m \]

Example 14: A man applies a force F = 300N at an angle of 60.0° to the door, 2.00 m from the hinges. Find the torque on the door, choosing the position of the hinges as the axis of rotation.

\[ \tau_F = rF \sin \theta = (2.00m)(300N) \sin 60.0^\circ \]

\[ = (2.00m)(260N) = 520N \cdot m \]

Suppose a wedge is placed 1.50 m from the hinges on the other side of the door. What minimum force must the wedge exert so that the force applied in part (a) won’t open the door?

\[ \tau_{hinge} + \tau_{wedge} + \tau_F = 0 \]

\[ 0 + F_{wedge} (1.50m) \sin(-90.0^\circ) + 520N \cdot m = 0 \]

\[ F_{wedge} = 347N \]
1. A boy and a girl are riding on a merry-go-round which is turning at a constant rate. The boy is near the outer edge, and the girl is closer to the center. Both will have the same non-zero angular velocity.

2. A boy and a girl are riding on a merry-go-round which is turning at a constant rate. The boy is near the outer edge, and the girl is closer to the center. The boy will have the greater translational velocity.

3. A boy and a girl are riding on a merry-go-round which is turning at a constant rate. The boy is near the outer edge, and the girl is closer to the center. The boy will have the greater centripetal acceleration.

4. A boy and a girl are riding on a merry-go-round which is turning at a constant rate. The boy is near the outer edge, and the girl is closer to the center. They have different speeds, but the same angular velocity.

5. A boy and a girl are riding on a merry-go-round that is turning. The boy is twice as far as the girl from the merry-go-round’s center. If the boy and girl are of equal mass, the boy’s moment of inertia is 4 times the girl’s with respect to the axis of rotation.

6. Angular velocity is expressed in units of radians per second.

7. A non-zero net torque will cause a change in angular velocity.

8. An ice skater is in a spin with her arms outstretched. If she pulls in her arms her kinetic energy increases.

9. An ice skater is in a tight spin. She stretches out her arms and slows down. In doing so, her angular momentum is conserved.

10. An ice skater is in a tight spin. She stretches out her arms and slows down. In doing so, her linear momentum is conserved.

11. An ice skater is in a tight spin. She stretches out her arms and slows down. In doing so, her kinetic energy is conserved.

12. An ice skater performs a pirouette (a fast spin) by pulling in her outstretched arms close to the body. Her moment of inertia decreases about the axis of rotation.
13. An ice skater performs a pirouette (a fast spin) by pulling in her outstretched arms close to the body. Her angular momentum does not change about the axis of rotation.

14. An ice skater performs a pirouette (a fast spin) by pulling in her outstretched arms close to the body. Her rotational kinetic energy increases about the axis of rotation.

15. We can best understand how a diver is able to control his rate of rotation while in the air (and thus enter the water in a vertical position) by observing that while in the air, his angular momentum is constant.

16. Two uniform solid spheres have the same mass, but one has twice the radius of the other. The ratio of the larger sphere’s moment of inertia to that of the smaller sphere is 4.

17. Consider two uniform solid spheres where one has twice the mass and twice the diameter of the other. The ratio of the larger moment of inertia to that of the smaller moment of inertia is 8.

18. Consider two uniform solid spheres where both have the same diameter, but one has twice the mass of the other. The ratio of the larger moment of inertia to that of the smaller moment of inertia is 2.