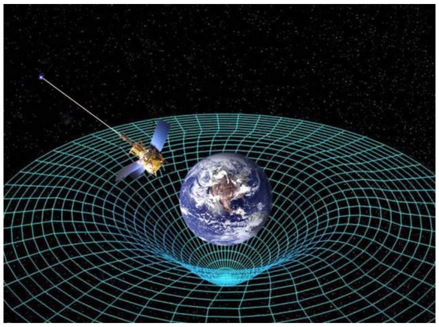
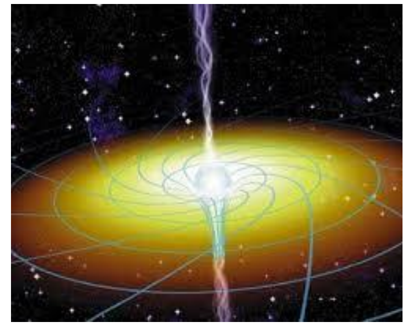


CHAPTER 26
The Special Theory of Relativity



Units

- **Galilean-Newtonian Relativity**
- **Postulates of the Special Theory of Relativity**
- **Simultaneity**
- **Time Dilation and the Twin Paradox**
- **Length Contraction**
- **Four-Dimensional Space-Time**
- **Relativistic Momentum and Mass**
- **The Ultimate Speed**
- **$E = mc^2$; Mass and Energy**
- **Relativistic Addition of Velocities**
- **The Impact of Special Relativity**

Basic Problems

- **The speed of every particle in the universe always remains *less than* the speed of light**
- **Newtonian Mechanics is a limited theory**
  - It places no upper limit on speed
  - It is contrary to modern experimental results
  - Newtonian Mechanics becomes a specialized case of Einstein's Theory of Special Relativity
    - When speeds are much less than the speed of light

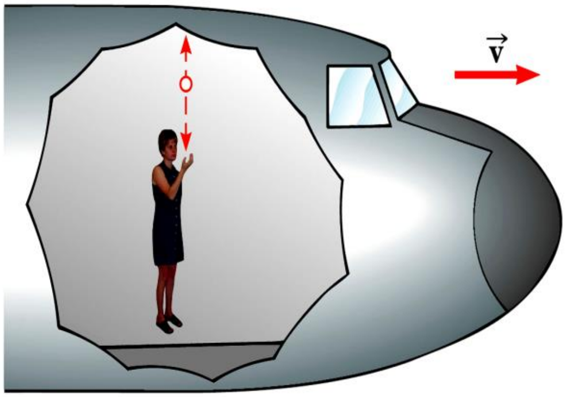
Foundation of Special Relativity

- **Reconciling of the measurements of two observers moving relative to each other**
  - Normally observers measure different speeds for an object
  - Special relativity relates two such measurements

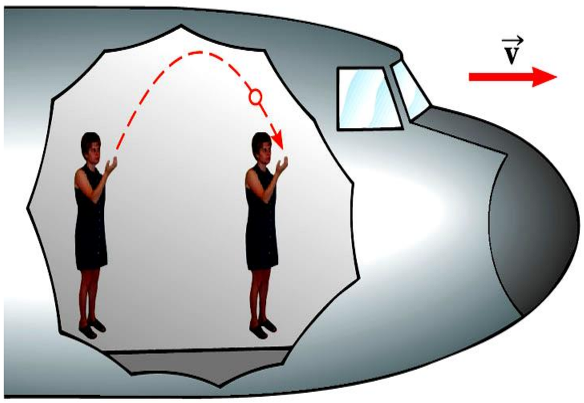
Galilean Relativity

- **Choose a *frame of reference***
  - Necessary to describe a physical event
- **According to Galilean Relativity, the laws of mechanics are the same in all inertial frames of reference**
  - An inertial frame of reference is one in which Newton's Laws are valid
  - Objects subjected to no forces will move in straight lines

Galilean Relativity – Example



- **A passenger in an airplane throws a ball straight up**
  - It appears to move in a vertical path
  - This is the same motion as when the ball is thrown while at rest on the Earth
  - The law of gravity and equations of motion under uniform acceleration are obeyed

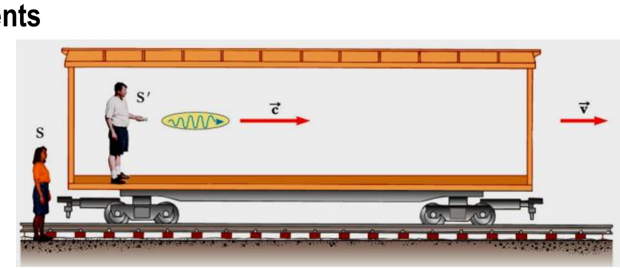


- **There is a stationary observer on the ground**
  - Views the path of the ball thrown to be a parabola
  - The ball has a velocity to the right equal to the velocity of the plane

- The two observers disagree on the shape of the ball's path
- Both agree that the motion obeys the law of gravity and Newton's laws of motion
- Both agree on how long the ball was in the air
- *Conclusion:* There is no preferred frame of reference for describing the laws of mechanics

Galilean Relativity – Limitations

- **Galilean Relativity does *not* apply to experiments in electricity, magnetism, optics, and other areas**
- **Results do not agree with experiments**
  - The observer should measure the speed of the pulse as v+c
  - Actually measures the speed as c



The Postulates of Special Relativity

The postulates of relativity as stated by Einstein:

1. **Equivalence of Physical Laws**

The laws of physics are the same in all inertial frames of reference.

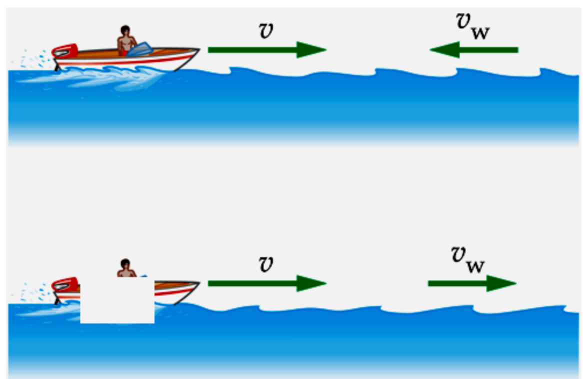
2. **Constancy of the Speed of Light**

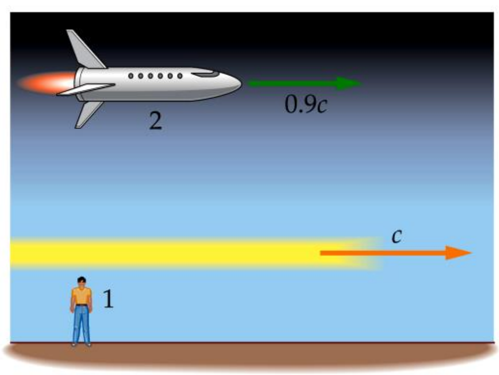
The speed of light in a vacuum, $c = 3.00 \times 10^8$ m/s, is the same in all inertial frames of reference, independent of the motion of the source or the receiver.

The first postulate is certainly reasonable; it would be hard to discover the laws of physics if it were not true!
But why would the speed of light be constant? It was thought that, like all other waves, light propagated as a disturbance in some medium, which was called the ether. The Earth's motion through the ether should be detectable by experiment. Experiments showed, however, no sign of the ether.

Other experiments and measurements have been done, verifying that the speed of light is indeed constant in all inertial frames of reference.

With water waves, our measurement of the wave speed depends on our speed relative to the water:



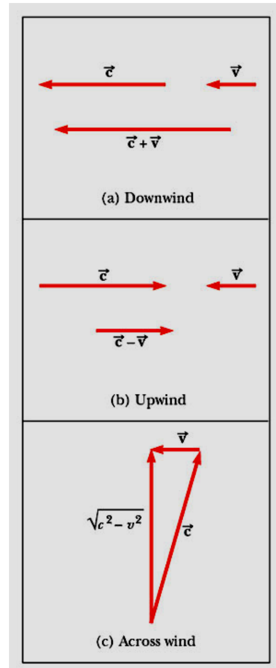


But with light, our measurements of its speed always give the same result:
The fact that the speed of light is constant also means that nothing can go faster than the speed of light – it is the ultimate speed limit of the universe.

Luminiferous Ether

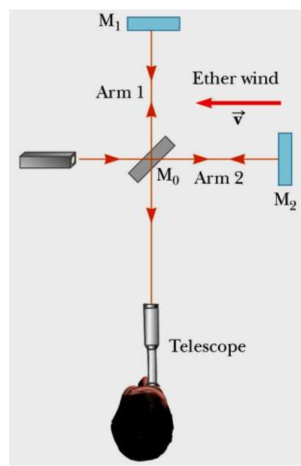
- **19th Century physicists compared electromagnetic waves to mechanical waves**
  - Mechanical waves need a medium to support the disturbance
- **The *luminiferous ether* was proposed as the medium required (and present) for light waves to propagate**
  - Present everywhere, even in empty space
  - Massless, but rigid medium
  - Could have no effect on the motion of planets or other objects

Verifying the Luminiferous Ether



- Associated with an ether was an *absolute frame* where the laws of e & m take on their simplest form
- Since the earth moves through the ether, there should be an “ether wind” blowing
- If v is the speed of the ether relative to the earth, the speed of light should have minimum (b) or maximum (a) value depending on its orientation to the “wind”

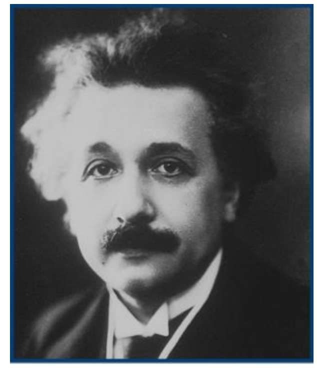
Michelson-Morley Experiment



- First performed in 1881 by Michelson
- Repeated under various conditions by Michelson and Morley
- Designed to detect small changes in the speed of light
  - By determining the velocity of the earth relative to the ether

- Used the Michelson Interferometer
- Arm 2 is aligned along the direction of the earth’s motion through space
- The interference pattern was observed while the interferometer was rotated through 90°
- The effect should have been to show small, but measurable, shifts in the fringe pattern
- Measurements failed to show any change in the fringe pattern
  - No fringe shift of the magnitude required was ever observed
- Light is now understood to be an electromagnetic wave, which requires no medium for its propagation
  - The idea of an ether was discarded
- The laws of electricity and magnetism are the same in all inertial frames
  - The addition laws for velocities were incorrect

Albert Einstein



- 1879 – 1955
- 1905 published four papers
  - 2 on special relativity
- 1916 published about General Relativity
- Searched for a unified theory
  - Never found one

Einstein's Principle of Relativity

- Resolves the contradiction between Galilean relativity and the fact that the speed of light is the same for all observers
- Postulates
  - **The Principle of Relativity**: All the laws of physics are the same in all inertial frames
  - **The constancy of the speed of light:** the speed of light in a vacuum has the same value in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light
- This is a sweeping generalization of the principle of Galilean relativity, which refers only to the laws of mechanics
- The results of *any kind* of experiment performed in a laboratory at rest must be the same as when performed in a laboratory moving at a constant speed past the first one
- No preferred inertial reference frame exists
- It is impossible to detect absolute motion

The Constancy of the Speed of Light

- Been confirmed experimentally in many ways
  - A direct demonstration involves measuring the speed of photons emitted by particles traveling near the speed of light
  - Confirms the speed of light to five significant figures
- Explains the null result of the Michelson-Morley experiment
- Relative motion is unimportant when measuring the speed of light
  - We must alter our common-sense notions of space and time

Consequences of Special Relativity

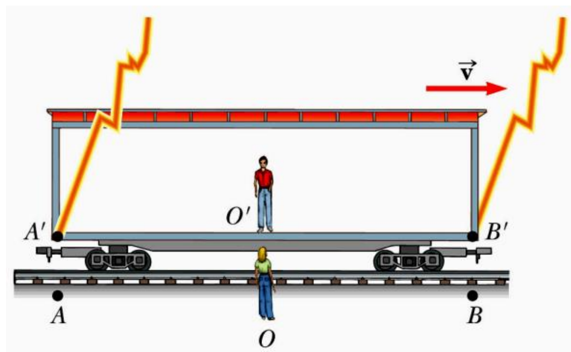
- Restricting the discussion to concepts of length, time, and simultaneity
- In relativistic mechanics
  - There is no such thing as absolute length
  - There is no such thing as absolute time
  - Events at different locations that are observed to occur simultaneously in one frame are not observed to be simultaneous in another frame moving uniformly past the first

Simultaneity

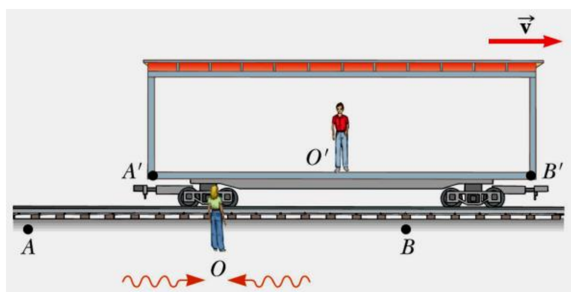
- In Special Relativity, Einstein abandoned the assumption of simultaneity
- Thought experiment to show this
  - A boxcar moves with uniform velocity
  - Two lightning bolts strike the ends
  - The lightning bolts leave marks (A' and B') on the car and (A and B) on the ground
  - Two observers are present: O' in the boxcar and O on the ground

Simultaneity – Thought Experiment Set-up

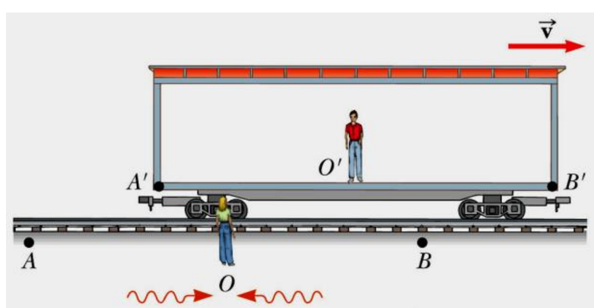
- Observer O is midway between the points of lightning strikes on the ground, A and B
- Observer O' is midway between the points of lightning strikes on the boxcar, A' and B'



- The light signals reach observer O at the same time
  - He concludes the light has traveled at the same speed over equal distances
  - Observer O concludes the lightning bolts occurred simultaneously



- By the time the light has reached observer O, observer O' has moved
- The light from B' has already moved by the observer, but the light from A' has not yet reached him
  - The two observers must find that light travels at the same speed
  - Observer O' concludes the lightning struck the front of the boxcar before it struck the back (they were not simultaneous events)

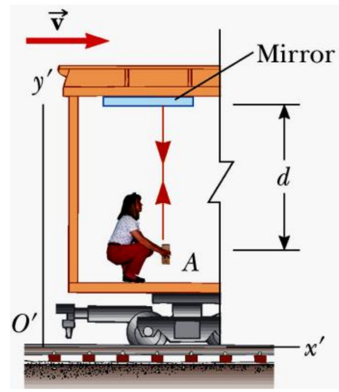


Simultaneity – Thought Experiment, Summary

- Two events that are simultaneous in one reference frame are in general not simultaneous in a second reference frame moving relative to the first

- That is, simultaneity is not an absolute concept, but rather one that depends on the state of motion of the observer
  - In the thought experiment, both observers are correct, because there is no preferred inertial reference frame

Time Dilation

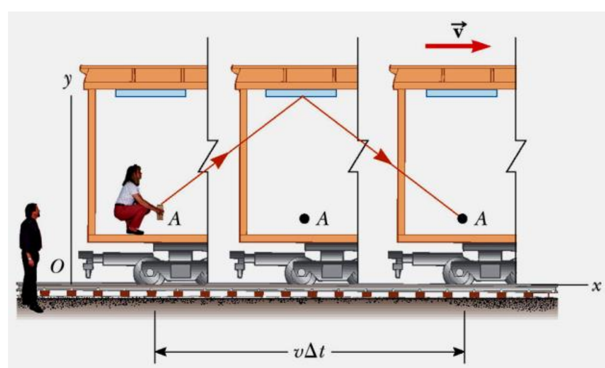


- The vehicle is moving to the right with speed v
- A mirror is fixed to the ceiling of the vehicle
- An observer, O', at rest in this system holds a laser a distance d below the mirror
- The laser emits a pulse of light directed at the mirror (event 1) and the pulse arrives back after being reflected (event 2)

Time Dilation, Moving Observer

- Observer O' carries a clock
- She uses it to measure the time between the events (Δt_p)
    - The p stands for proper
  - She observes the events to occur at the same place
  - Δt_p = distance/speed = (2d)/c

Time Dilation, Stationary Observer



- Observer O is a stationary observer on the earth
- He observes the mirror and O' to move with speed *v*
- By the time the light from the laser reaches the mirror, the mirror has moved to the right
- The light must travel farther with respect to O than with respect to O'

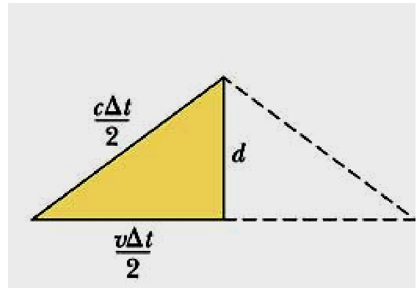
Time Dilation, Observations

- Both observers must measure the speed of the light to be c
- The light travels farther for O
- The time interval, Δt, for O is longer than the time interval for O', Δt_p

Time Dilation, Time Comparisons

$$\Delta t = \frac{\Delta t_p}{\sqrt{1 - v^2/c^2}} = \gamma \Delta t_p$$

$$where\ \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$



- Observer O measures a longer time interval than observer O’

Time Dilation, Summary

- The time interval Δt between two events measured by an observer moving with respect to a clock is longer than the time interval Δt_p between the same two events measured by an observer at rest with respect to the clock
- A clock moving past an observer at speed *v* runs more slowly than an identical clock at rest with respect to the observer by a factor of γ^{-1}

Identifying Proper Time

- The time interval Δt_p is called the *proper time*
  - The proper time is the time interval between events as measured by an observer who sees the events occur at the same position
    - You must be able to correctly identify the observer who measures the proper time interval

Alternate Views

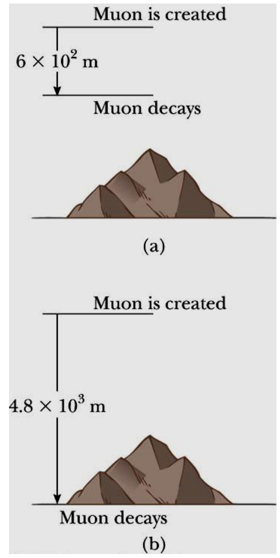
- The view of O’ that O is really the one moving with speed v to the left and O’s clock is running more slowly is just as valid as O’s view that O’ was moving
- The principle of relativity requires that the views of the two observers in uniform relative motion must be equally valid and capable of being checked experimentally

Time Dilation – Generalization

- All physical processes slow down relative to a clock when those processes occur in a frame moving with respect to the clock
  - These processes can be chemical and biological as well as physical
- Time dilation is a very real phenomena that has been verified by various experiments

Time Dilation Verification – Muon Decays

- Muons are unstable particles that have the same charge as an electron, but a mass 207 times more than an electron
- Muons have a half-life of Δt_p = 2.2μs when measured in a reference frame at rest with respect to them (a)
- Relative to an observer on earth, muons should have a lifetime of $\gamma\ \Delta t_p$ (b)
- A CERN experiment measured lifetimes in agreement with the predictions of relativity



Example 1: A car traveling 100 km/h covers a certain distance in 10.0 s according to the driver's watch. What does an observer at rest on Earth measure for the time interval?

$$\Delta t = \frac{\Delta t_o}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{10.00s}{\sqrt{1-\left(\frac{27.8m/s}{3.00x10^8 m/s}\right)^2}} = \frac{10.00s}{\sqrt{1-(8.59x10^{-15})}}$$

A calculator will give an answer of 10.00s. A computer will give $4x10^{-14}s$ more than 10.00 s.

Example 2: The period of a pendulum is measured to be 3.00 s in the inertial frame of the pendulum. What is the period as measured by an observer moving at a speed of 0.950c with respect to the pendulum?

$$\Delta t = \frac{\Delta t_p}{\sqrt{1-v^2/c^2}} = \frac{3.00s}{\sqrt{1-\frac{(0.950c)^2}{c^2}}} = 9.61s$$

The moving observer considers the pendulum to be moving, and moving clocks are observed to run more slowly.

The Twin Paradox – The Situation

- A thought experiment involving a set of twins, Speedo and Goslo
- Speedo travels to Planet X, 20 light years from earth
  - His ship travels at 0.95c
  - After reaching planet X, he immediately returns to earth at the same speed
- When Speedo returns, he has aged 13 years, but Goslo has aged 42 years

The Twins' Perspectives

- Goslo's perspective is that he was at rest while Speedo went on the journey
- Speedo thinks he was at rest and Goslo and the earth raced away from him on a 6.5 year journey and then headed back toward him for another 6.5 years
- The paradox – which twin is the traveler and which is really older?

The Twin Paradox – The Resolution

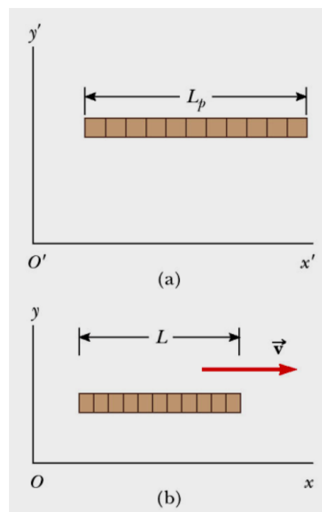
- Relativity applies to reference frames moving at uniform speeds
- The trip in this thought experiment is not symmetrical since Speedo must experience a series of accelerations during the journey
- Therefore, Goslo can apply the time dilation formula with a proper time of 42 years
  - This gives a time for Speedo of 13 years and this agrees with the earlier result
- There is no true paradox since Speedo is not in an inertial frame

Length Contraction

- The measured distance between two points depends on the frame of reference of the observer
- The *proper length*, L_p, of an object is the length of the object measured by someone at rest relative to the object
- The length of an object measured in a reference frame that is moving with respect to the object is always less than the proper length
  - This effect is known as *length contraction*

$$L = \frac{L_P}{\gamma} = L_P\sqrt{1 - \frac{v^2}{c^2}}$$

- Length contraction takes place only along the direction of motion



Four-Dimensional Space-Time

Space and time are even more intricately connected. Space has three dimensions, and time is a fourth. When viewed from different reference frames, the space and time coordinates can mix.

Example 3: A starship is measured to be 125 m long while it is at rest with respect to an observer. If the starship now flies past the observer at a speed of 0.99c, what length will the observer measure for the starship?

$$L = L_p\sqrt{1 - v^2 / c^2} = (125m)\sqrt{1-(0.99c)^2 / c^2} = 17.6m$$

Example 4: An observer on Earth sees a spaceship at an altitude of 4,350 km moving downward toward Earth with a speed of 0.970c. (a) What is the distance from the spaceship to Earth as measured by the spaceship's captain?

$$L = L_p\sqrt{1-v^2/c^2} = 4,350km\sqrt{1-(0.970c)^2/c^2} = 1.06x10^3 km$$

(b) After firing his engines, the captain measures his ship's altitude as 267 km, while the observer on Earth measures it to be 625 km. What is the speed of the spaceship at this instant?

$$L = L_p\sqrt{1-v^2/c^2}$$

$$L^2 = L_p^2(1-v^2/c^2) \rightarrow 1-v^2/c^2 = \left(\frac{L}{L_p}\right)^2$$

$$v = c\sqrt{1-(L/L_p)^2} = c\sqrt{1-(267km/625km)^2} = 0.904c$$

Relativistic Definitions

- To properly describe the motion of particles within special relativity, Newton's laws of motion and the definitions of momentum and energy need to be generalized
- These generalized definitions reduce to the classical ones when the speed is much less than c

Relativistic Momentum

- To account for conservation of momentum in all inertial frames, the definition must be modified
  - $$p \equiv \frac{mv}{\sqrt{1-v^2/c^2}} = \gamma mv$$
  - v is the speed of the particle, m is its mass as measured by an observer at rest with respect to the mass
  - When v << c, the denominator approaches 1 and so p approaches mv

Relativistic Addition of Velocities

- Galilean relative velocities cannot be applied to objects moving near the speed of light
- Einstein's modification is
$$v_{ab} = \frac{v_{ad} + v_{db}}{1 + \frac{v_{ad}v_{db}}{c^2}}$$
  - The denominator is a correction based on length contraction and time dilation

Relativistic Corrections

- Remember, relativistic corrections are needed because *no material objects can travel faster than the speed of light*

A basic result of special relativity is that nothing can equal or exceed the speed of light. This would require infinite momentum – not possible for anything with mass.

Example 5: Suppose that Kirk's spacecraft is traveling at 0.600c in the positive x-direction, as measured by a nearby observer, while Scotty is traveling in his own vehicle directly toward Kirk in the negative x-direction at -0.800c relative the nearby observer. What's the velocity of Kirk relative to Scotty?

$$v_{KS} = \frac{v_{KS} + v_{SO}}{1 + \frac{v_{KS} v_{SO}}{c^2}}$$

K = Kirk S = Scotty
O = Observer

$$0.600c = \frac{v_{KS} - 0.800c}{1 + \frac{v_{KS}(-0.800c)}{c^2}} \rightarrow \left(1 - \frac{0.800 v_{KS}}{c}\right) 0.600c = v_{KS} - 0.800c$$

$$0.600c - 0.480 v_{KS} = v_{KS} - 0.800c$$

$$v_{KS} = 0.946c$$

$E = mc^2$; Mass and Energy

At relativistic speeds, not only is the formula for momentum modified; that for energy is as well. The total energy can be written:

$$E = \gamma m_0 c^2 = \frac{m_0 c^2}{\sqrt{1 - v^2/c^2}}$$

Where the particle is at rest,

$$E_0 = m_0 c^2$$

Combining the relations for energy and momentum gives the relativistic relation between them:

$$E^2 = p^2 c^2 + m_0^2 c^4$$

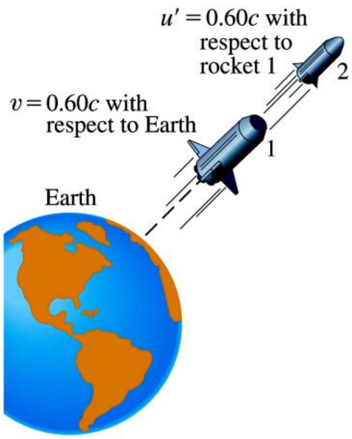
All the formulas presented here become the usual Newtonian kinematic formulas when the speeds are much smaller than the speed of light.
There is no rule for when the speed is high enough that relativistic formulas must be used – it depends on the desired accuracy of the calculation.

Example 6: When two moles of hydrogen and one mole of oxygen react to form two moles of water, the energy released is 484 kJ. How much does the mass decrease in this reaction?

$$\Delta m = \frac{\Delta E}{c^2} = \frac{(-484x10J)}{(3.00x10^8 m/s)^2} = -5.38x10^{-12} kg$$

The initial mass of the system is 0.002 kg + 0.016 kg = 0.018 kg. Thus the change in mass is relatively very tiny and can normally be neglected.

Relativistic Addition of Velocities



Relativistic velocities cannot simply add; the speed of light is an absolute limit. The relativistic formula is:

$$u = \frac{v + u'}{1 + vu'/c^2}$$

Example 7: calculate the speed of rocket 2.

$$u = \frac{0.60c + 0.60c}{1 + \frac{(0.60c)(0.60c)}{c^2}} = \frac{1.20c}{1.36} = 0.88c$$

Relativistic Energy

- The definition of kinetic energy requires modification in relativistic mechanics
- KE = $\gamma mc^2 - mc^2$
  - The term mc^2 is called the *rest energy* of the object and is independent of its speed
  - The term γmc^2 is the *total energy*, E, of the object and depends on its speed and its rest energy

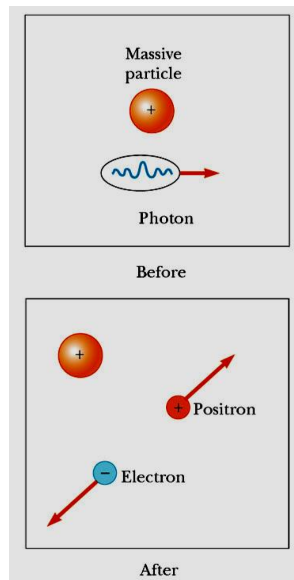
Relativistic Energy – Consequences

- A particle has energy by virtue of its mass alone
  - A stationary particle with zero kinetic energy has an energy proportional to its inertial mass
- The mass of a particle may be completely convertible to energy and pure energy may be converted to particles

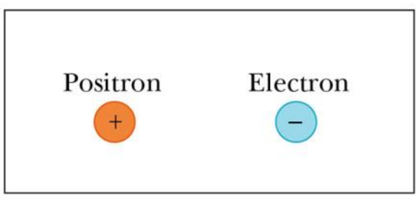
Energy and Relativistic Momentum

- It is useful to have an expression relating total energy, E, to the relativistic momentum, p
  - $E^2 = p^2c^2 + (mc^2)^2$
    - When the particle is at rest, p = 0 and $E = mc^2$
    - Massless particles (m = 0) have E = pc
  - This is also used to express masses in energy units
    - Mass of an electron = 9.11×10^{-31} kg = 0.511 Me
    - Conversion: 1 u = 931.494 MeV/c^2

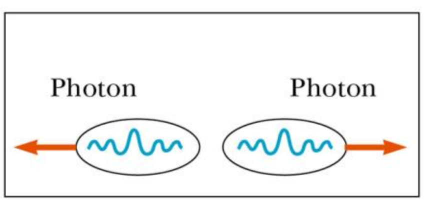
Pair Production



- An electron and a positron are produced and the photon disappears
  - A positron is the antiparticle of the electron, same mass but opposite charge
- Energy, momentum, and charge must be conserved during the process
- The minimum energy required is $2m_e$ = 1.02 MeV



Before



After

- In pair annihilation, an electron-positron pair produces two photons
  - The inverse of pair production
- It is impossible to create a single photon
  - Momentum must be conserved

Mass – Inertial vs. Gravitational

- Mass has a gravitational attraction for other masses

$$F_g = G\frac{m_g m_g}{r^2}$$

- Mass has an inertial property that resists acceleration
  - $F_i = m_i a$
- The value of G was chosen to make the values of m_g and m_i equal

Einstein's Reasoning Concerning Mass

- That m_g and m_i were directly proportional was evidence for a basic connection between them
- No mechanical experiment could distinguish between the two
- He extended the idea to no experiment of any type could distinguish the two masses

Postulates of General Relativity

- All laws of nature must have the same form for observers in any frame of reference, whether accelerated or not
- In the vicinity of any given point, a gravitational field is equivalent to an accelerated frame of reference without a gravitational field
  - This is the *principle of equivalence*

Implications of General Relativity

- Gravitational mass and inertial mass are not just proportional, but completely equivalent
- A clock in the presence of gravity runs more slowly than one where gravity is negligible
- The frequencies of radiation emitted by atoms in a strong gravitational field are shifted to lower frequencies
  - This has been detected in the spectral lines emitted by atoms in massive stars
- A gravitational field may be "transformed away" at any point if we choose an appropriate accelerated frame of reference – a freely falling frame
- Einstein specified a certain quantity, the *curvature of spacetime*, that describes the gravitational effect at every point

Curvature of Spacetime

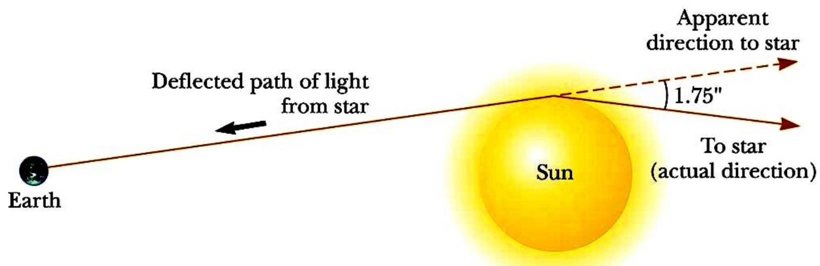
- There is no such thing as a gravitational force
  - According to Einstein
- Instead, the presence of a mass causes a curvature of spacetime in the vicinity of the mass
  - This curvature dictates the path that all freely moving objects must follow

General Relativity Summary

- Mass one tells spacetime how to curve; curved spacetime tells mass two how to move
  - John Wheeler's summary, 1979
- The equation of general relativity is roughly a proportion:

Average curvature of spacetime a energy density

  - The actual equation can be solved for the *metric* which can be used to measure lengths and compute trajectories



- General Relativity predicts that a light ray passing near the Sun should be deflected by the curved spacetime created by the Sun's mass
- The prediction was confirmed by astronomers during a total solar eclipse
- Explanation of Mercury's orbit
  - Explained the discrepancy between observation and Newton's theory
- Time delay of radar bounced off Venus
- Gradual lengthening of the period of binary pulsars due to emission of gravitational radiation

Black Holes

- If the concentration of mass becomes great enough, a black hole is believed to be formed
- In a black hole, the curvature of space-time is so great that, within a certain distance from its center, all light and matter become trapped
- The radius is called the *Schwarzschild radius*
  - Also called the *event horizon*
  - It would be about 3 km for a star the size of our Sun
- At the center of the black hole is a *singularity*
  - It is a point of infinite density and curvature where spacetime comes to an end