We make use of the laws of conservation of linear momentum and of energy to analyze collisions. The law of conservation of momentum is particularly useful when dealing with a system of two or more objects that interact with each other, such as in collisions.

**Momentum**

- The linear momentum $p$ of an object of mass $m$ moving with a velocity $v$ is defined as the product of the mass and the velocity
  - $p = mv$
  - SI Units are kg m / s
  - Vector quantity, the direction of the momentum is the same as the velocity’s

**Momentum and Its Relation to Force**

Momentum is a vector symbolized by the symbol $p$, and is defined as $p = m\vec{v}$

The rate of change of momentum is equal to the net force: $\sum F = \frac{\Delta p}{\Delta t}$

This can be shown using Newton’s second law.

**Example 1**: For a top player, a tennis ball may leave the racket on the serve with a speed of 55 m/s. If the ball has a mass of 0.060 kg and is in contact with the racket for about 4 ms, estimate the average force on the ball. Would this force be large enough to lift a 60-kg person?

$$F = \frac{\Delta p}{\Delta t} = \frac{mv_2 - mv_1}{\Delta t} = \frac{(0.060\text{kg})(55\text{m/s}) - 0}{0.004\text{s}} \approx 800\text{N}$$

$$\approx (60\text{kg})(9.80\text{m/s}^2) = 600\text{N}$$

Needed to lift the person. (Yes, enough to lift a person)
**Example 2:** Water leaves a hose at a rate of 1.5kg/s with a speed of 20 m/s and is aimed at the side of a car, which stops it. What is the force exerted by the water on the car?

\[
F = \frac{\Delta p}{\Delta t} = \frac{p_{\text{final}} - p_{\text{initial}}}{\Delta t} = \frac{0 - 30 \text{kg} \cdot \text{m/s}}{1.0 \text{s}} = -30 \text{N}
\]

**Conservation of Momentum**

During a collision, measurements show that the total momentum does not change:

\[
m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B
\]

More formally, the law of conservation of momentum states:

**The total momentum of an isolated system of objects remains constant.**

**Example 3:** A 10,000-kg railroad car A, traveling at a speed of 24.0 m/s strikes an identical car B, at rest. If the cars lock together as a result of the collision, what is their common speed just afterward? \[
m_A \vec{v}_A + m_B \vec{v}_B = m_A \vec{v}'_A + m_B \vec{v}'_B
\]

\[
v' = \frac{m_A}{m_A + m_B} v_A = \frac{10,000 \text{kg}}{(10,000 \text{kg} + 10,000 \text{kg})} (24.0 \text{m/s})
\]

To the right, mutual speed is ½ of car A.
Example 4: Calculate the recoil velocity of a 5.0-kg rifle that shoots a 0.020-kg bullet at a speed of 620 m/s.

\[
m_Bv_B + m_Rv_R = m_Bv_B' + m_Rv_R'
\]

\[
0 + 0 = m_Bv_B' + m_Rv_R'
\]

\[
v_R = -\frac{m_Bv_B'}{m_R} = -\frac{(0.020\text{kg})(620\text{m/s})}{(5.0\text{kg})} = -2.5\text{m/s}
\]

Momentum conservation works for a rocket as long as we consider the rocket and its fuel to be one system, and account for the mass loss of the rocket.

Collisions and Impulse

During a collision, objects are deformed due to the large forces involved.

Since \( \vec{F} = \frac{\Delta \vec{p}}{\Delta t} \), we can write \( \vec{F} \Delta t = \Delta \vec{p} \)

The definition of impulse: \( \text{Impulse} = \vec{F} \Delta t \)

Since the time of the collision is very short, we need not worry about the exact time dependence of the force, and can use the average force.
The impulse tells us that we can get the same change in momentum with a large force acting for a short time, or a small force acting for a longer time.

This is why you should bend your knees when you land; why airbags work; and why landing on a pillow hurts less than landing on concrete.

- The air bag increases the time of the collision
- It will also absorb some of the energy from the body
- It will spread out the area of contact
  - decreases the pressure
  - helps prevent penetration wounds

**Example 5:** A golf ball with a mass of 0.05kg is struck with a club. The force on the ball varies from zero when contact is made up to some maximum value and then back to zero when the ball leaves the club. If the ball leaves the club at a velocity of +44m/s find the following:

(a) the magnitude of the impulse due to the collision.

\[ I = \Delta p = p_f - p_i \]

\[ = (0.05 \text{kg})(44 \text{m/s} - 0) = +2.2 \text{kg} \cdot \text{m/s} \]

(b) Estimate the duration of the collision and the average force acting on the ball.

\[ \Delta t = \frac{\Delta x}{v} = \frac{0.02 \text{m}}{22 \text{m/s}} = 9.1 \times 10^{-4} \text{s} \quad \text{Displacement = radius of ball about 0.02 m.} \]

\[ F = \frac{\Delta p}{\Delta t} = \frac{2.2 \text{kg} \cdot \text{m/s}}{9.1 \times 10^{-4} \text{s}} = +2.4 \times 10^3 \text{ N} \]

**Conservation of Energy and Momentum in Collisions**

Momentum is conserved in all collisions. Collisions in which kinetic energy is conserved as well are called elastic collisions, and those in which it is not are called inelastic.
Elastic Collisions in One Dimension

Here we have two objects colliding elastically. We know the masses and the initial speeds. Since both momentum and kinetic energy are conserved, we can write two equations. This allows us to solve for the two unknown final speeds.

Inelastic Collisions

With inelastic collisions, some of the initial kinetic energy is lost to thermal or potential energy. It may also be gained during explosions, as there is the addition of chemical or nuclear energy. A completely inelastic collision is one where the objects stick together afterwards, so there is only one final velocity.

Example 6: In a crash test, a car of mass 1500 kg collides with a wall and rebounds. The initial and final velocities of the car are $v_i = -15 \text{ m/s}$ and $v_f = 2.60 \text{ m/s}$ respectively, if the collisions lasts for 0.150 s, find:
(a) the impulse delivered to the car due to the collision.
\[ p_i = mv_i = (1500 \text{ kg})(-15.0 \text{ m/s}) = -2.25 \times 10^4 \text{ kg} \cdot \text{m/s} \]
\[ p_f = mv_f = (1500 \text{ kg})(+2.60 \text{ m/s}) = +0.390 \times 10^4 \text{ kg} \cdot \text{m/s} \]
\[ I = p_f - p_i = +0.390 \times 10^4 \text{ kg} \cdot \text{m/s} - (-2.25 \times 10^4 \text{ kg} \cdot \text{m/s}) \quad I = 2.64 \times 10^4 \text{ kg} \cdot \text{m/s} \]

(b) The size and direction of the force exerted on the car.
\[ F = \frac{\Delta p}{\Delta t} = \frac{2.64 \times 10^4 \text{ kg} \cdot \text{m/s}}{0.150 \text{ s}} = +1.76 \times 10^5 \text{ N} \]

**Example 7:** An SUV with mass 1800 kg is traveling eastbound at +15.0 m/s, collides with a compact car with mass 900 kg traveling westbound at -15.0 m/s.
(a) Find the change in the velocity of each car.
\[ m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f \]
\[ (1800 \text{ kg})(15.0 \text{ m/s}) + (900 \text{ kg})(-15.0 \text{ m/s}) = (1800 \text{ kg} + 900 \text{ kg}) v_f \]
\[ v_f = +5.00 \text{ m/s} \]

(b) Find the change in the velocity of each car:
\[ \Delta v_1 = v_f - v_{1i} = (5.00 \text{ m/s} - 15.0 \text{ m/s}) = -10.0 \text{ m/s} \]
\[ \Delta v_2 = v_f - v_{2i} = 5.00 \text{ m/s} - (-15.0 \text{ m/s}) = +20.0 \text{ m/s} \]

(c) Find the change in the KE of the system consisting of both cars.
\[ KE_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} (1800 \text{ kg})(15.0 \text{ m/s})^2 + \frac{1}{2} (900 \text{ kg})(-15.0 \text{ m/s})^2 = 3.04 \times 10^5 \text{ J} \]
\[ KE_f = \frac{1}{2} (m_1 + m_2) v_f^2 = \frac{1}{2} (1800 \text{ kg} + 900 \text{ kg})(5.00 \text{ m/s})^2 = 3.38 \times 10^4 \text{ J} \]
\[ \Delta KE = KE_f - KE_i = -2.70 \times 10^5 \text{ J} \]

**Example 8:** The ballistic pendulum is a device used to measure the speed of a fast-moving projectile such as a bullet. The bullet is fired into a large block of wood suspended from some light wires. The bullet is stopped by the block, and the entire system swings up to a height h.
It is possible to obtain the initial speed of the bullet by measuring h and the two masses. As an example, assume that the mass of the bullet, m1, is 5.00g, the mass of the pendulum, m2, is 1.0kg, and h = 5.00cm. Find the initial speed of the bullet, v1.

\[ m_1v_{i1} + m_2v_{i2} = (m_1 + m_2)v_f \]
\[ (5.00 \times 10^{-3} \text{ kg})v_i + 0 = (1.005 \text{ kg})v_f \]
\[ v_f = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(0.05 \text{ m})} = 0.990 \text{ m/s} \]
\[ v_i = \frac{(1.005 \text{ kg})(0.990 \text{ m/s})}{5.00 \times 10^{-3} \text{ kg}} = 199 \text{ m/s} \]

**Collisions in Two or Three Dimensions**

Conservation of energy and momentum can also be used to analyze collisions in two or three dimensions, but unless the situation is very simple, the math quickly becomes unwieldy.

Here, a moving object collides with an object initially at rest. Knowing the masses and initial velocities is not enough; we need to know the angles as well in order to find the final velocities.

**Rocket Propulsion**

- The operation of a rocket depends on the law of conservation of momentum as applied to a system, where the system is the rocket plus its ejected fuel
  - This is different than propulsion on the earth where two objects exert forces on each other
    - road on car
    - train on track
- The rocket is accelerated as a result of the thrust of the exhaust gases
- This represents the inverse of an inelastic collision
  - Momentum is conserved
  - Kinetic Energy is increased (at the expense of the stored energy of the rocket fuel)

- The initial mass of the rocket is \( M + \Delta m \)
  - \( M \) is the mass of the rocket
  - \( m \) is the mass of the fuel
- The initial velocity of the rocket is
• The rocket’s mass is \(M\)
• The mass of the fuel, \(\Delta m\), has been ejected
• The rocket’s speed has increased to \(\mathbf{V} + \Delta \mathbf{V}\)

The basic equation for rocket propulsion is:

\[ v_f - v_i = v_e \ln \left( \frac{M_i}{M_f} \right) \]

– \(M_i\) is the initial mass of the rocket plus fuel
– \(M_f\) is the final mass of the rocket plus any remaining fuel
– The speed of the rocket is proportional to the exhaust speed

**Thrust of a Rocket**

• The thrust is the force exerted on the rocket by the ejected exhaust gases
• The instantaneous thrust is given by

\[ Ma = M \frac{\Delta v}{\Delta t} = v_e \frac{\Delta M}{\Delta t} \]

– The thrust increases as the exhaust speed increases and as the burn rate \((\Delta M/\Delta t)\) increases

**Example 9:** A rocket has a total mass of \(1 \times 10^5\,\text{kg}\) and a burnout mass of \(1 \times 10^4\,\text{kg}\), including engines, shell, and payload. The rocket blasts off from Earth and exhausts all its fuel in 4.00 min. burning the fuel at a steady rate with an exhaust velocity of \(v_e = 4.50 \times 10^3\,\text{m/s}\).

(a) If air friction and gravity are neglected, what is the speed of the rocket at burnout?

\[ v_f = v_i + v_e \ln \left( \frac{M_i}{M_f} \right) \]

\[ = 0 + (4.5 \times 10^3\,\text{m/s}) \ln \left( \frac{1.00 \times 10^5\,\text{kg}}{1.00 \times 10^4\,\text{kg}} \right) = 1.04 \times 10^4\,\text{m/s} \]

(b) What thrust does the engine develop at liftoff?

\[ Ma = \Sigma F = F_r - M g \]

\[ a = \frac{F_r}{M} - g = \frac{1.69 \times 10^6\,\text{N}}{1.00 \times 10^4\,\text{kg}} - 9.80\,\text{m/s}^2 = 7.10\,\text{m/s}^2 \]
(d) Estimate the speed at burnout if gravity isn’t neglected.

\[ \Delta v_g = -g \Delta t = -(9.80 \text{ m/s}^2)(2.40 \times 10^2 \text{ s}) = -2.35 \times 10^3 \text{ m/s} \]

\[ v_f = 1.04 \times 10^4 \text{ m/s} - 2.35 \times 10^3 \text{ m/s} = 8.05 \times 10^3 \text{ m/s} \]

**Center of Mass**

In (a), the diver’s motion is pure translation; in (b) it is translation plus rotation. There is one point that moves in the same path a particle would take if subjected to the same force as the diver. This point is called the center of mass (CM).

The general motion of an object can be considered as the sum of the translational motion of the CM, plus rotational, vibrational, or other forms of motion about the CM.

For two particles, the center of mass lies closer to the one with the most mass:

\[ x_{CM} = \frac{m_A x_A + m_B x_B}{m_A + m_B} = \frac{m_A x_A + m_B x_B}{M} \]

where \( M \) is the total mass.
The center of gravity is the point where the gravitational force can be considered to act. It is the same as the center of mass as long as the gravitational force does not vary among different parts of the object.

**CM for the Human Body**

High jumpers have developed a technique where their CM actually passes under the bar as they go over it. This allows them to clear higher bars.

**Center of Mass and Translational Motion**

This is particularly useful in the analysis of separations and explosions; the center of mass (which may not correspond to the position of any particle) continues to move according to the net force.
CHAPTER 7
LINEAR MOMENTUM

CONCEPTS

1. In a game of pool, the white cue ball hits the #5 ball and stops while the #5 ball moves away with the same velocity as the cue ball had originally. This type of collision is elastic.

2. Kinetic energy is conserved when it is an elastic collision.

3. Two objects move toward each other collide, and separate. If there was no net external force acting on the objects, but some kinetic energy was lost, then the collision was not elastic and total linear momentum was conserved.

4. Two objects collide and bounce off each other. Kinetic energy is conserved only if the collision is elastic.

5. Kinetic energy is never conserved for a perfectly inelastic collision free of external forces.

6. Two objects collide and stick together. Kinetic energy is definitely not conserved.

7. Two objects collide and stick together. Linear momentum is definitely conserved.

8. The product of an object’s mass and velocity is equal to momentum.

9. When a cannon fires a cannonball, the cannon will recoil backward because the momentum of the cannonball and cannon is conserved.
10. A 100-kg football linebacker moving at 2 m/s tackles head-on an 80-kg halfback running 3 m/s. Neglecting the effects due to digging in of cleats, the halfback will drive the linebacker backwards.

11. A small car meshes with a large truck in a head-on collision. The small car and large truck experience the same average force.

12. A small object collides with a large object and sticks. Both objects experience the same magnitude of momentum change.

13. When two cars collide and lock together both momentum and total energy is conserved.

14. A golf ball moving east at a speed of 4 m/s, collides with a stationary bowling ball. The golf ball bounces back to the west, and the bowling ball moves very slowly to the east. Neither object experiences the greater magnitude impulse, they are the same.

15. The area under the curve on an F – t graph represents impulse.

16. Two equal mass balls, A and B, are dropped from the same height, and rebound off the floor. The A ball rebounds to a higher position. The A ball is subjected to the greater magnitude impulse during its collision with the floor.

17. If an object is acted on by a non-zero net external force, its momentum will not remain constant.
18. A rubber ball and a lump of putty have equal mass. They are thrown with equal speed against a wall. The ball bounces back with nearly the same speed with which it hit. The putty sticks to the wall. The ball experiences the greater momentum.

\[ m_b v_b + m_p v_p = m_b v_b' + m_p v_p' \]

19. A baseball catcher wears a glove rather than just using bare hands to catch a pitched baseball because the force on the catcher’s hand is reduced because the glove increases the time of impact.

20. In a baseball game, a batter hits a ball for a home run. Compared to the magnitude of the impulse imparted to the ball, the magnitude of the impulse imparted to the bat is the same.

21. A freight car moves along a frictionless level railroad track at constant speed. The car is open on top. A large load of sand is suddenly dumped into the car. This causes the velocity of the car to decrease.

22. For an object on the surface of the earth, the center of gravity and the center of mass are the same point.

23. Tightrope walkers walk with a long flexible rod in order to lower their center of mass.

24. The center of gravity of an object may be thought of as the “balance point.”

25. A plane, flying horizontally, releases a bomb, which explodes before hitting the ground. Neglecting air resistance, the center of mass of the bomb fragments, just after the explosion moves along a parabolic path.